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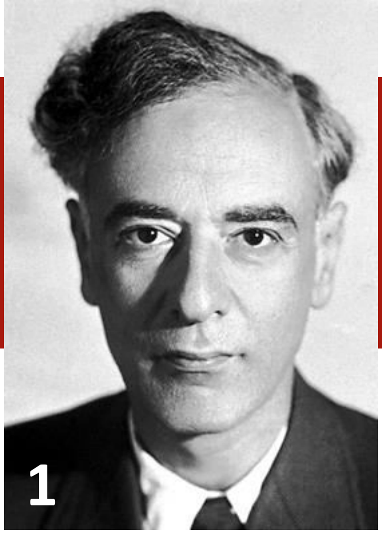
Topologiczne ciecze kwantowe

1. **Wstęp** – topologiczne stany materii
2. **Teoria FQHE** – złożone fermiony
3. **Obliczenia numeryczne** – diagonalizacja hamiltonianu (configuration interaction)
4. **Wyniki/publikacje**

Spotkanie sprawozdawcze WCSS (PWr), 11.02.2014

Introduction

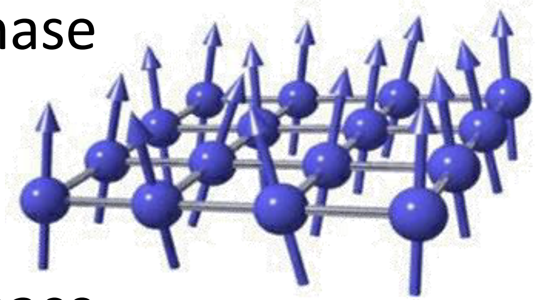


Particles + interactions → emergent order

Ordered phases known until 1980 – **spontaneous symmetry breaking** (Landau[1] theory)

<u>Ordered phase:</u>

- Low-temperature phase (ground state) has **lower symmetry** than high-temperature phase
- **local order parameter**



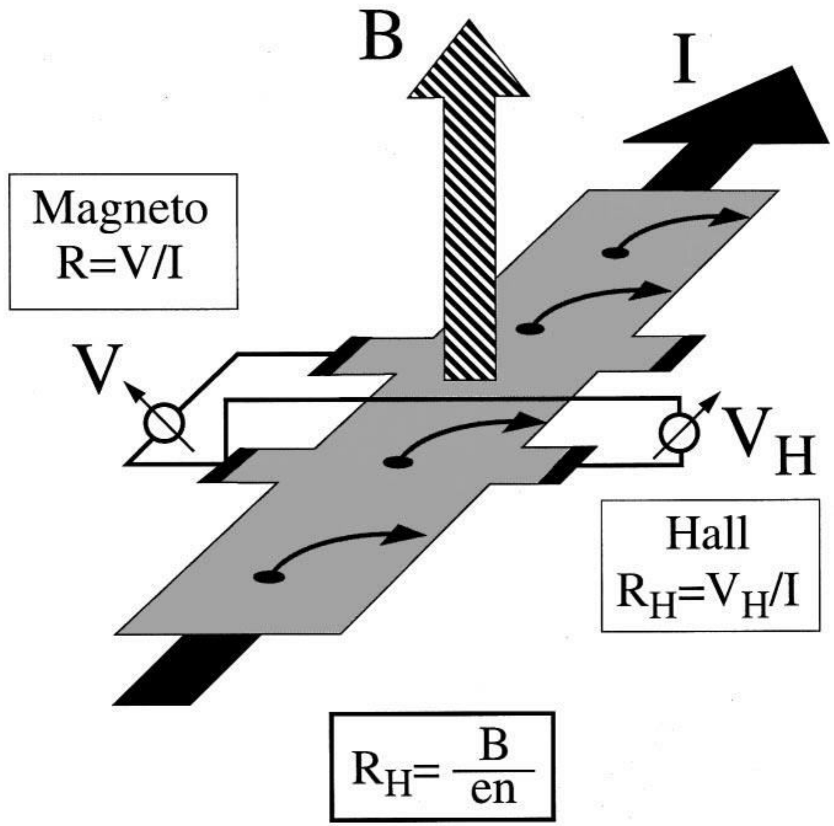
<u>Examples of symmetry breaking:</u>

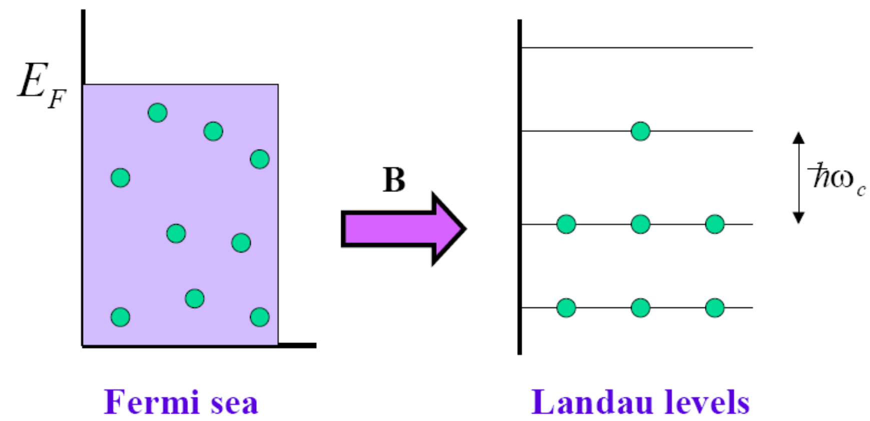
- Crystal – translations/rotations of empty space
- Magnet – rotations in spin space, time reversal

QHE

*other/related topological states :
- topological insulators (HgTe, Bi_2Se_3)
- topological superconductors (Sr_2RuO_4)

5.2.1980^{2am} – **quantum Hall effect**
(Klaus von Klitzing[1]) – **topological phase***





Landau level degeneracy per area:
$\varrho_\phi = N_\phi/A = B/\phi_0;\ \ \phi_0 = hc/e$

Filling factor: $\nu = N/N_\phi = \varrho/\varrho_\phi$

QHE

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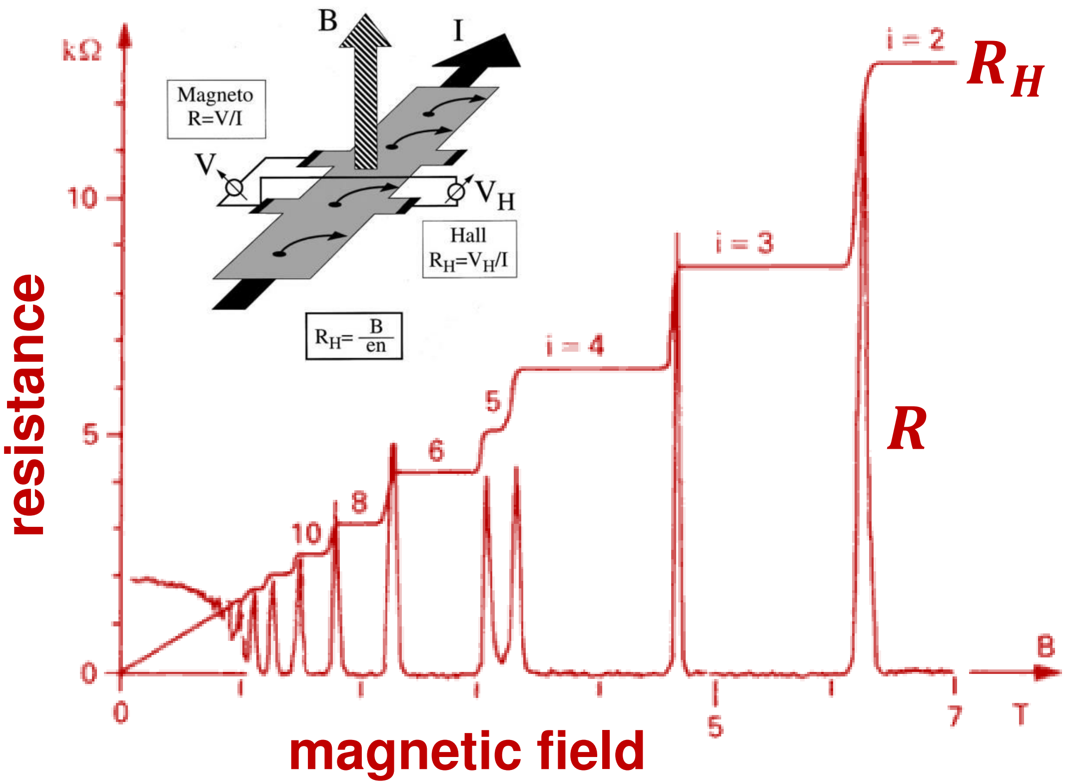

$$R_H \sim \rho_{xy} = (\sigma_{xy})^{-1}$$

$$\boldsymbol{\sigma_{xy} = i\, e^2/h}$$

quantization 3:10^{10}

Also: $\boldsymbol{R = 0}$

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QHE

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- topological insulators (HgTe, Bi_2Se_3)
- topological supercond


1

5.2.1980[2am] – **quantum Hall effe**
(Klaus von Klitzing[1]) – **topologic**

- ground state has **higher symmetry** than high-temperature phase
- **nonlocal order parameter**
  Chern number = ∫ Berry curvature

Wave vector $\boldsymbol{k}$ in closed loop →
Berry phase of Bloch w-fun $u_m(\boldsymbol{k})$:
$\oint A_m \, ds_k$; $A_m \equiv i \langle u_m | \nabla_k | u_m \rangle$

Berry curvature: $\mathcal{F}_m = \nabla_k \times A_m$

Stokes theorem:
$$\oint A_m \; ds_k = \iint \mathcal{F}_m \, d^2k$$
Chern number (~total curv):
$$n_m = (2\pi)^{-1} \iint_{occ} \mathcal{F}_m \, d^2k$$
Gauss & Bonnet (1848):
$$\iint K dA + \int k_g ds = 2\pi \, \chi_M$$
For a closed surface:
$$\oiint K \; dA = 2\pi \, (2 - 2g)$$
For torus (2D Brillouin zone):
$$g = 1 \quad \Rightarrow \quad n = 0$$
Thouless, Kohmoto, Nightingale, den Nijs (1982):
$$\sigma_{xy} = \boldsymbol{n} e^2/h \text{ (over occup. LLs)}$$

QHE

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5.2.1980^{2am} – **quantum Hall effect** (Klaus von Klitzing[1]) – **topological phase***

- ground state has **higher symmetry** than high-temperature phase
- **nonlocal order parameter**
  Chern number = ∫ Berry curvature
- properties depend on **topological invariant** (e.g. σ_{xy} ~ Chern number)

Exact quantization of σ_{xy} is **independent** of:
- material
- type of structure
- sample geometry
- disorder
- magnetic field
- temperature
- …

QHE

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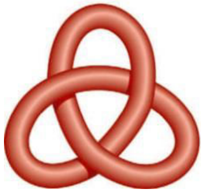

- ground state has **higher symmetry** than high-temperature phase
- **nonlocal order parameter**
  Chern number = ∫ Berry curvature
- properties depend on **topological invariant** (e.g. σ_{xy} ~ Chern number)
- **topologically non-trivial** full band
  → insulating interior, edge currents

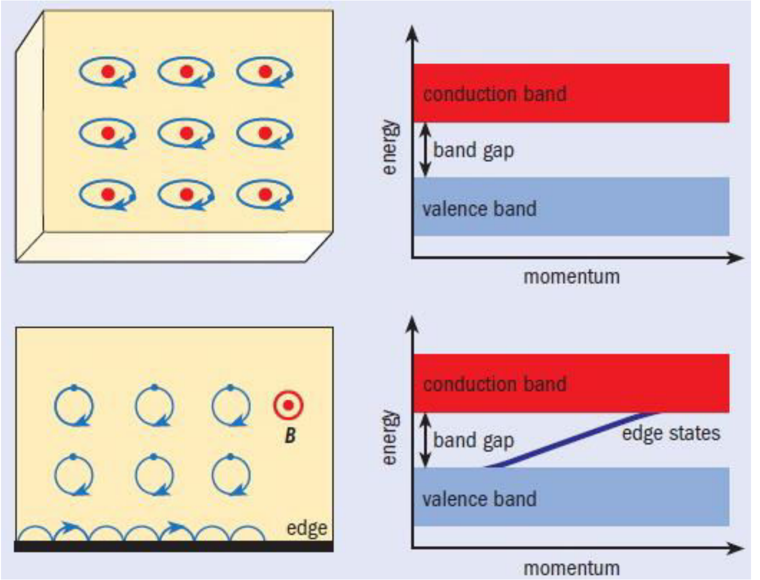

Fig: Kane & Moore, Physics World 2011

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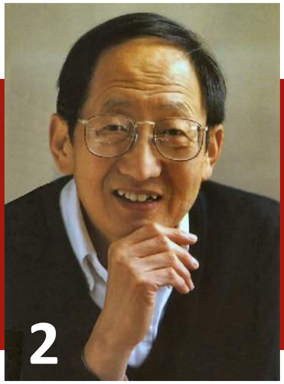
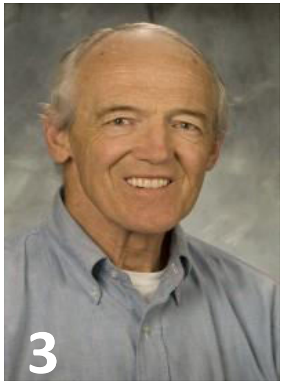
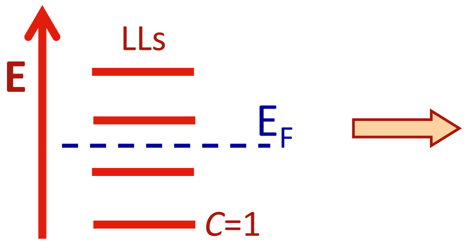
FQHE

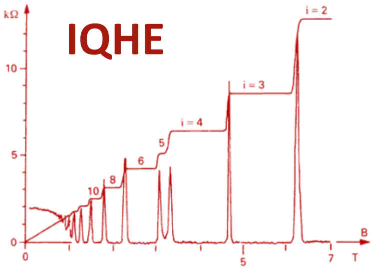
*other/related states :
- quantum spin glasses
- fractional Chern insulator

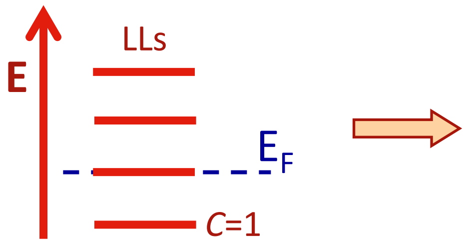




1982 – **fractional quantum Hall effect**
(H L Störmer[1], D C Tsui[2], A C Gossard[3])
– **„fractional" topological phase***







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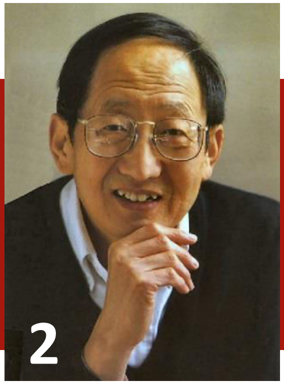
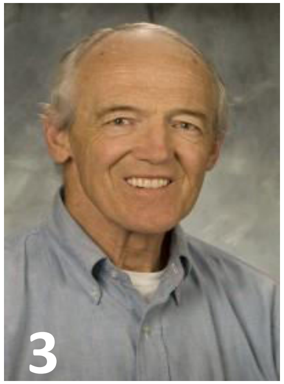
Fermi level inside LL
(fractional LL filling)

FQHE

*other/related states :
- quantum spin glasses
- fractional Chern insulator





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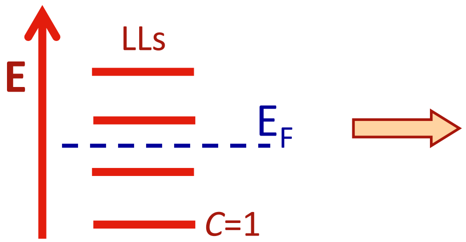
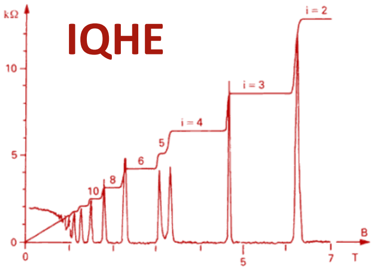
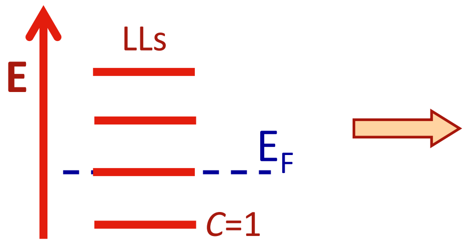
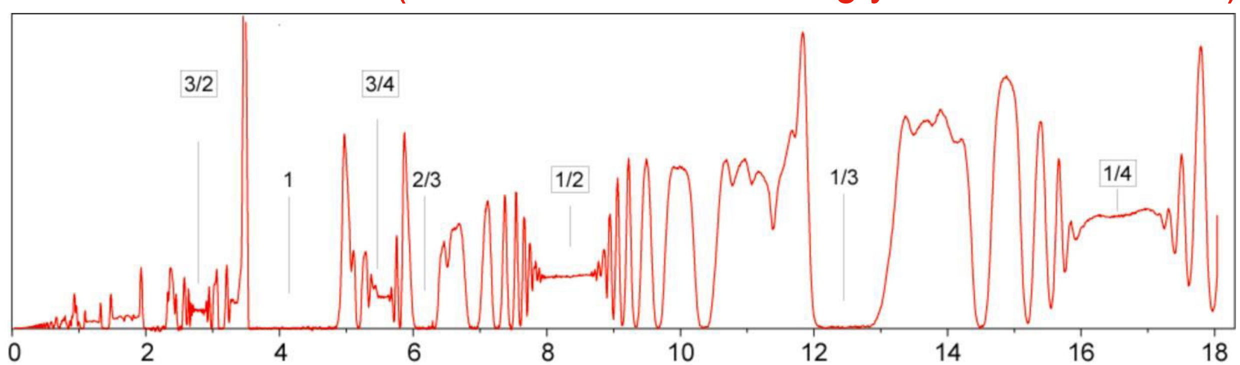




(similar behavior; strongly correlated states)





Fermi level inside LL
(fractional LL filling)

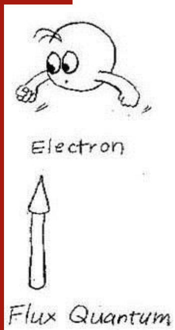
J. K. Jain, PRL (1989)

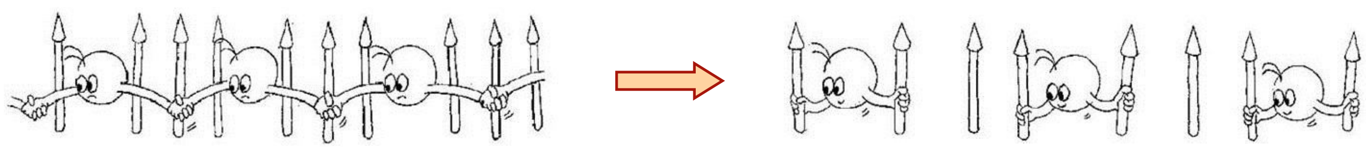
Composite fermions

2D electron gas + magnetic field
→ widely spaced degenerate Landau levels
→ partially filled, isolated LL

composite fermion (CF) = electron + correlation hole
= *e* + 2p vortices of many-body wave function
= *e* + 2p magnetic flux quanta *hc/e*

interaction → correlations → **emergence of (essentially) free quasiparticles**





(Fig. Kwon Park)

interacting electrons in strong field *B* almost free CFs in reduced field *B**

$$\Phi^* = \Phi - N \cdot 2p \cdot hc/e; \quad B^* = B - \left(2p \cdot hc/e\right)\rho; \quad \left(\nu^*\right)^{-1} = \nu^{-1} - 2p$$

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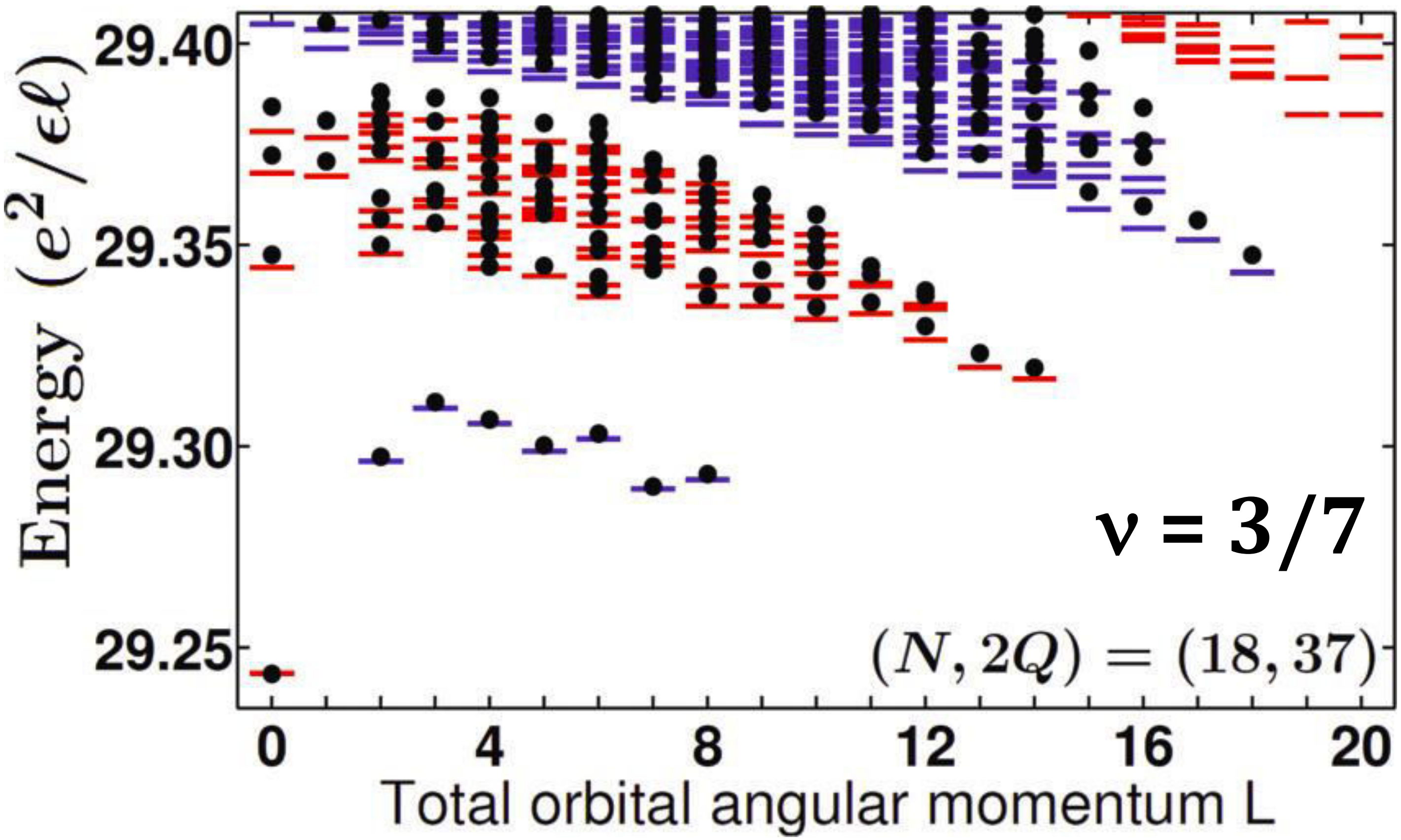
Numerical tests of CFs



$\nu = 3/7$

dim=387M
dim(L)~1M

N electrons on sphere; radial magnetic field (Dirac monopole 2*Q*)
2*Q* = magnetic flux through surface; LL degener. = 2*Q*+1; ν ~ *N*/2*Q*

What are CFs?

- **Collective** – involving many (all) electrons
- **Topological** – vorticity 2p
- **Nearly free** – true quasiparticles of FQHE
- **Different from Laughlin QPs** (and more accurate)
- **Unify IQHE with FQHE** (e.g., n* = CF Chern number)
- **One particle** (same for all fractions)
- **Predict experiments**
  - correct fractions, relative strengths of $\nu = n/(2pn \pm 1)$
  - dispersions of collective modes (CF excitons)
  - CF Fermi sea at B*=0 (ν=1/even)
  - spin physics: magnetism, transitions, coll. modes, skyrmions

Configuration interaction

Many-electron Hamiltonian

$$H = \sum_i \frac{1}{2\mu}\left(\mathbf{p}_i - \frac{e}{c}\mathbf{A}_i\right)^2 + \sum_{i<j} V\left(\mathbf{r}_i - \mathbf{r}_j\right)$$

High magnetic field →
large cyclotron gap
fractional LL occupation

Model extended 2DEG by *N*<∞
→ **Haldane spherical geometry**
2D symmetry (rotations)
radial field B
(Dirac monopole Φ=2Q)
LL = shell of ℓ = Q
LL degeneracy = 2Q+1

Single particle states:
monopole harmonics $|1,m\rangle$

Interaction:
e^2/r, chord distance
(or model repulsions)

CI basis:
***N*-electron determinants**

$$c_1^+ \ldots c_N^+ |\mathrm{vac}\rangle$$

Hamiltonian (2-body → sparse)

$$H_{2-body} = \sum_{ijkl} V_{ij;kl}\, c_i^+ c_j^+ c_k c_l$$

$$V_{ij;kl} = \sum_m C_{ij}^m C_{kl}^m\, V(m)$$

Lanczos diagonalization → *E*(*L*)

Role of Exciton Screening in the 7/3 Fractional Quantum Hall Effect

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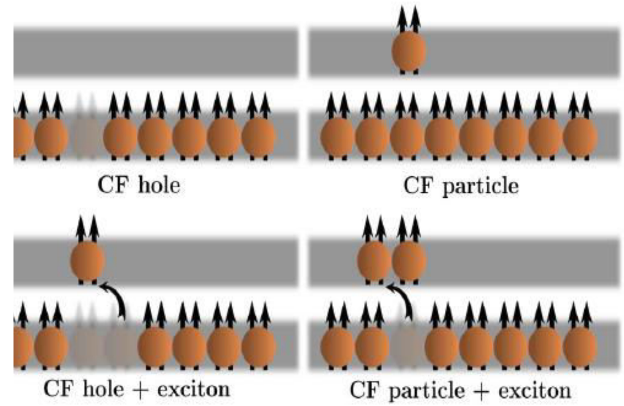
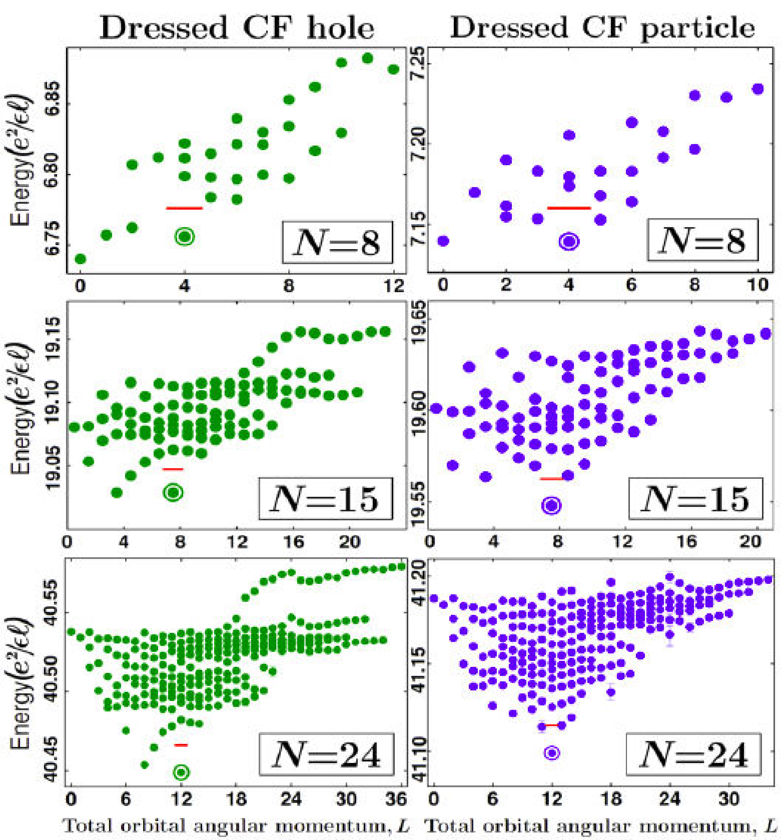
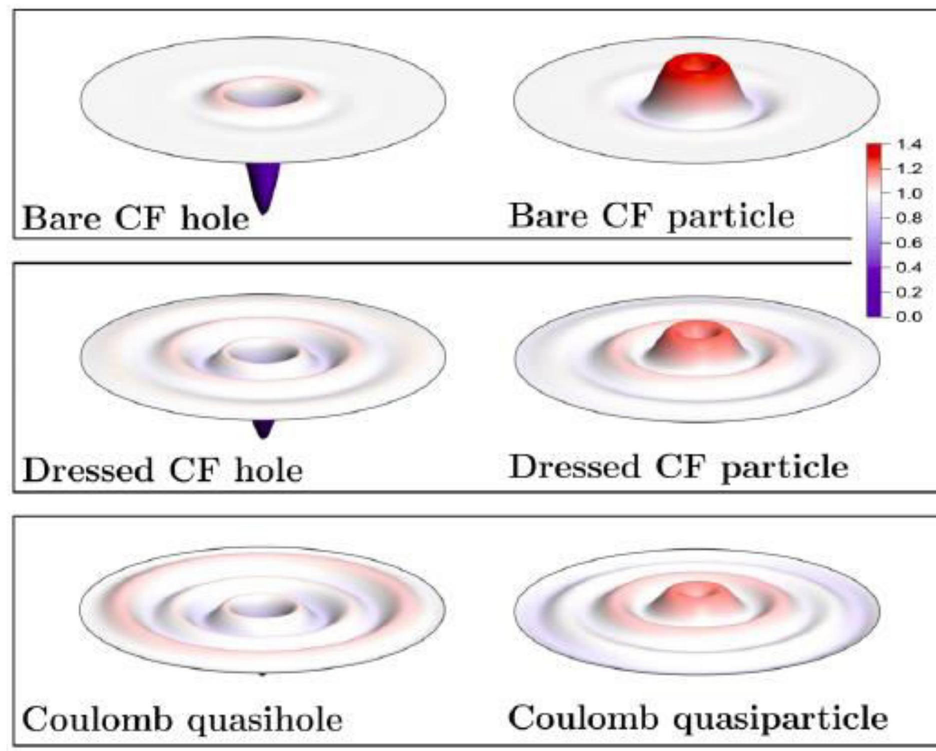
The excitations of the 7/3 fractional Hall state, one of the most prominent states in the second Landau level, are not understood. We study the effect of screening by composite fermion excitons and find that it causes a strong renormalization at 7/3, thanks to a relatively small exciton gap and a relatively large residual interaction between composite fermions. The excitations of the 7/3 state are to be viewed as composite fermions dressed by a large exciton cloud. Their wide extent has implications for experiments as well as for analysis of finite system exact diagonalization studies.

DOI: 10.1103/PhysRevLett.110.186801 PACS numbers: 73.43.−f, 05.30.Pr, 71.10.Pm







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Enigmatic 4/11 State: A Prototype for Unconventional Fractional Quantum Hall Effect

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State counting for excited bands of the fractional quantum Hall effect: Exclusion rules for bound excitons

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Tripartite composite fermion states

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